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**Transition from  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  Representation to  
 $SU(2)_L \times U(1)_Y$  by  $q$ -Deformation and the  
Corresponding Classical Breaking Term of Chiral  $U(2)$**

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**Abstract**

The minimal Standard Model exhibits a nontrivial chiral  $U(2)$  symmetry if the VEV and the hypercharge splitting  $\Delta = (y_R^u - y_R^d)/2$  of right-handed leptons (quarks) in a family vanish and  $Q = T_0 + Y$  independently in each helicity sector. As a generalization, we start with  $SU(2)_L \times SU(2)_R \times U(1)_{(B-L)}$  and introduce  $\Delta$  as a continuous parameter which is a measure of explicit symmetry breakdown. Values  $0 \leq \Delta \leq 1/2$  take the neutral generator of the isospin- $\frac{1}{2}$  representation to the singlet representation, i.e. ‘deforms’ the LR representation into the minimal Standard one. The corresponding classical  $O(3)$ -breaking term is a magnetic field perpendicular to the  $x_3$ -axis. A simple mapping on the fundamental Drinfeld-Jimbo  $q$ -deformed  $SU(2)$  representation is given.

# 1. Introduction

The very underlying symmetry of a system is the largest nontrivial symmetry that can be obtained in any limiting set of values of the parameters of the system. This symmetry refers to the minimal structure that occurs in the symmetric phase and may be broken at an arbitrary set of values of its parameters. The electroweak Standard Model (SM) of one fermion family exhibits a chiral  $U(2)$ ,

$$U(2)_L \times U(2)_R = SU(2)_L \times U(1)_{Y_L} \times SU(2)_R \times U(1)_{Y_R}. \quad (1)$$

The  $U(1)$  charges are arranged such that the electric charge is given as a combination of the generators in the Cartan subalgebra of each chirality sector:\*

$$Q_x = T_{0x} + Y_x; \quad x = L, R. \quad (2)$$

The  $SU(2)_R$  factor is hidden in the fermion representation of the minimal SM:  $T_R \equiv 0$ . Therefore,  $T_{0R} \equiv 0$  and we have a splitting

$$\Delta \equiv \frac{y_R^u - y_R^d}{2} \quad (3)$$

of hypercharges  $y^{u,d}$  in the right-handed sector of potential isospin- $\frac{1}{2}$  components with a charge matrix

$$Y_R = Q = \begin{pmatrix} y_R^u & 0 \\ 0 & y_R^d \end{pmatrix}. \quad (4)$$

Clearly, eqn. (1) and (2) are not unique as any hidden (trivial) factor can in principle be added to the standard  $SU(2) \times U(1)$  symmetry and to

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\* Projectors  $\frac{1}{2}(1 \pm \gamma_5)$  fix normalization of  $Q$  in  $J_{em} = \bar{\Psi}\gamma^\mu Q\Psi$  such that  $Q_L = Q_R = Q = \text{diag}(0, -1)$  [ $\text{diag}(2/3, -1/3)$ ] for leptons [quarks] and we use  $T_0 = \frac{1}{2}\tau_3$ ,  $T_\pm = \frac{1}{\sqrt{2}}(\tau_1 \pm i\tau_2)$ , where  $\tau_i$  are Pauli matrices normalized to  $\tau^2 = 1$ . These choices avoid further factors 1/2 in quantum numbers.

$Q$  and only the requirement of eq. (2) makes the  $SU(2)_R$  definitely appear in eq. (1). The search for the origin of electroweak symmetry breakdown is the search for the underlying symmetry and for the parameters which are nonzero in the broken phase.

One crucial quantity in the symmetry breakdown is the splitting  $\Delta_g = g_u - g_d$  of Yukawa-couplings  $g_{u,d}$  within an isospin doublet. Finite  $\Delta_g$  breaks  $SU(2)$  down to  $U(1)$ , but  $\Delta_g$  can hardly be regarded independent from either the VEV  $v$  or  $\Delta$ : the renormalization procedure yields Yukawa-couplings with  $U(1)$  corrections such that even possibly degenerated tree-level couplings are always infected with  $SU(2)$ -violating self-energies. Therefore  $\Delta_g$  will not vanish in the physical lagrangian at an arbitrary scale. Masses  $m$  indeed show a numerical connection to charge, which has recently been put into an empirical formula for  $m_i$  across the families  $i = 1, 2, 3$  [1]:

$$\frac{m_2}{m_1} = 3 \left( \frac{m_3}{m_2} \right)^{\frac{3}{2}|Q|}. \quad (5)$$

The reason for this behaviour with  $Q$  is unknown and it can actually also be hypercharge that is involved, eq. (4). The factor  $3/2$  may however point to some deviation from  $Q$  in the origin of  $SU(2)$  violations. Various ways to connect  $\Delta$  and  $\Delta_g$  are possible: pure radiative effects [2, 3] or direct strong fermionic coupling to hypercharge currents [4] can be used. The existence of fermion mixing however favors gauge eigenstates, not mass eigenstates to be important in the true mechanism. Eq. (5) states the importance of the  $U(1)$  factor for the mass generation mechanism: it determines horizontal as well as vertical mass gaps in the standard spectrum. It also tells us that the relevant quantity for isospin violation is already known and does not have to come from beyond the SM. At the same time, as Yukawa-couplings are only meaningful once chirality is broken, the responsible interaction may not show maximal parity violation [5], but must possess a (nontrivial) diagonal subgroup.

The largest nontrivial chiral symmetry we get as a limit of the standard  $SU(2)_L \times U(1)_Y$  is (1): We require eq. (2) and take

$$v \rightarrow 0, \quad \Delta \rightarrow 0. \quad (6)$$

This yields (1) just in the representation of the much explored left-right symmetric model (LR);  $SU(2)_R$  is not hidden and  $Y_L = Y_R = \frac{1}{2}(B - L)$  [6]. Parity will be broken spontaneously by  $SU(2)$  couplings  $g_L \neq g_R$  or VEV's  $v_L \neq v_R$  and should be restored in the symmetric phase at higher energies. At  $\Delta = 0$  however,  $Y$  cannot introduce a vertical mass gap in eq. (5) and chirality breaking is also suppressed if one chirality sector is hidden. From this point of view, more general values of  $\Delta$  are interesting.

If the underlying symmetry of the SM is broken explicitly, it is interesting to ask for a physical term for this breaking in the lagrangian also irrespective of any details of the mass spectrum. If e.g. the SM has a nontrivial  $U(2)_L \times U(2)_R$  symmetric limit, but parity is broken not only in low energy states, a breaking term should be the corresponding physical sector that stands for initial symmetry breakdown.

The purpose of this paper is to construct a class of algebras that includes the LR representation as one limit and the minimal SM representation as the other and to calculate the classical symmetry breaking term. In section 2 we generalize the  $SU(2)$  algebra by introducing  $\Delta$  as a continuous parameter and show that the corresponding  $O(3)$ -breaking term is a magnetic field perpendicular to the  $x_3$ -axis direction and proportional to  $\Delta$ . In section 3 we show the connection to the well-known  $SU(2)_q$  by Drinfeld and Jimbo in a simple mapping between  $\Delta$ - and  $q$ -representations. Section 4 consists of a remark on the freedom of introducing a further parameter in the solution of the classical system and section 5 is the summary.

## 2. General $\Delta$ and the Classical $SU(2)_R$ Breaking Term

We consider the well-known Left-Right Model [6] and the Standard Model with no Higgs sector coupled to the fermion-boson content. The parameter that connects LR and SM representations  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  and  $SU(2)_L \times U(1)_Y$  is the hypercharge-splitting eq. (3) taking continuous values

$$0 \leq \Delta \leq \frac{1}{2}.$$

To break the symmetry, we perform a non-orthogonal  $GL(2)$  transformation in the right-handed Cartan subalgebra of the LR model to obtain a generalized basis  $(\bar{Y}, \bar{T}_0)$ , which only satisfies eq. (2):

$$\begin{pmatrix} \bar{Y} \\ \bar{T}_0 \end{pmatrix} = \begin{pmatrix} 1 & 2\Delta \\ 0 & 1 - 2\Delta \end{pmatrix} \begin{pmatrix} \frac{1}{2}(B - L) \\ T_0 \end{pmatrix}. \quad (7)$$

The LR model chooses  $\Delta = 0$  while in the SM  $\Delta = 1/2$ .  $(B - L)$  is anomaly free and so is  $\bar{Y}$ : the relevant sums of triangle diagrams involving 1 to 3 external  $U(1)$  gauge fields are proportional to  $\text{Tr}(\{\bar{T}_i, \bar{T}_j\}\bar{Y})$ ,  $\text{Tr}(\bar{T}_i\bar{Y}^2)$  and  $\text{Tr}(\bar{Y}^3)$ . Using eq. (2) isolates pure vectors, which have powers only of  $Q$ , but by eq. (7) the cancelation is seen to work within each family:

$$\begin{aligned} \text{Tr}(T_0^3) &= 0; \\ \text{Tr}[T_0^2(B - L)] &= \text{Tr}(B - L) = \frac{N_q \cdot N_c}{3} - N_l; \\ \text{Tr}[T_0(B - L)^2] &\sim \text{Tr}(T_0) = 0; \\ \text{Tr}[(B - L)^3] &= N_q N_c \left(\frac{1}{3}\right)^3 - N_l, \end{aligned} \quad (8)$$

where  $N_q$  ( $N_l$ ) is the number of quarks (leptons). The left-handed sector is identical to the SM one and, like in the SM, the abelian factor is gauged only in the vector subgroup, so that the last of eqn. (8) does not contribute.

The symmetry under consideration is now

$$SU(2)_L \times U(1)_{(B-L)_L} \times \left[ SU(2)_R \times U(1)_{(B-L)_R} \right]_{\Delta}. \quad (9)$$

$\bar{T}_0$  is fixed and the question is how to close the  $[SU(2)_R]_{\Delta}$  algebra.  $[SU(2)_R]_{\Delta}$  should contain the charged sector of the LR model as  $\Delta \rightarrow 0$ , but is otherwise unspecified.

In a simple isotropic renormalization of  $SU(2)$  generators

$$\bar{T}_0 = (1 - 2\Delta)T_0, \quad \bar{T}_{\pm} = \sqrt{1 - 2\Delta}T_{\pm}, \quad (10)$$

all  $\Delta$ -dependence could be absorbed into the (primordial) gauge coupling,

$$g_R \rightarrow \bar{g}_R(\Delta) \equiv (1 - 2\Delta)g_R. \quad (11)$$

$SU(2)_R$  gauge transformations would then read

$$D_{\mu}\Psi_R \rightarrow \exp[i\bar{g}_R T_j \omega(x)_R^j] D_{\mu}\Psi_R, \quad (12)$$

$$\bar{\Psi}_R \rightarrow \bar{\Psi}_R \exp[-i\bar{g}_R T_j \omega(x)_R^j], \quad (13)$$

where  $D_{\mu} = \partial_{\mu} + i\bar{g}_R T_j W_{\mu}^j$  and we have

$$[\bar{Y}, T_0] = 0, \quad [\bar{Y}, T_{\pm}] = \pm 2\Delta T_{\pm}. \quad (14)$$

As long as the  $B_{\mu}$  gauge field behaves trivially under  $SU(2)_R$ , the hypercharge interaction term  $J_{\bar{Y}_R}^{\mu} B_{\mu} = \bar{\Psi}_R [\frac{1}{2}(B - L) + 2\Delta T_0] \gamma^{\mu} \Psi_R B_{\mu}$  is not invariant:

$$i\bar{\Psi}_R \gamma^{\mu} \bar{Y} \Psi_R B_{\mu} \rightarrow i\bar{\Psi}_R \gamma^{\mu} \bar{Y} \Psi_R B_{\mu} - \bar{g}_R \bar{\Psi}_R \gamma^{\mu} [\bar{Y}, T_j \omega^j] \Psi_R B_{\mu}, \quad (15)$$

because of the  $T_0$  term in  $\bar{Y}$ . It is proportional to  $\bar{g}_R \cdot \Delta$ , which vanishes at both  $\Delta = 0$  and  $\Delta = 1/2$ , i.e. in LR and SM representations, and is maximal at  $\Delta = 1/4$ . The first of eqn. (14) ensures the  $U(1)$  to survive in the breakdown

$$U(2)_R = SU(2)_R \times U(1)_{Y_R} \xrightarrow{\Delta \neq 0} U(1)_{Y_R} \times U(1)_{T_0}. \quad (16)$$

We are interested in explicit breaking of chiral  $U(2)$  to the SM  $SU(2)_L \times U(1)_Y$  and a corresponding physical term in the Lagrangian. Therefore let us now consider a classical system and generalize the ordinary  $SU(2)$ -algebra with commutation relations

$$[T_0, T_\pm] = \pm T_\pm, \quad [T_+, T_-] = T_0 \quad (17)$$

and Casimir

$$C = 2 T_+ T_- + T_0(T_0 - 1) = 2 T_- T_+ + T_0(T_0 + 1) \equiv T(T + 1). \quad (18)$$

Eq. (7) together with leaving  $T_\pm$  unchanged can be taken as a deforming map on the fundamental  $SU(2)$  representation [8] and closing  $\bar{T}_0$  with  $T_\pm$  gives

$$[\bar{T}_0, T_\pm] = \pm(1 - 2\Delta) T_\pm, \quad [T_+, T_-] = (1 - 2\Delta)^{-1} \bar{T}_0. \quad (19)$$

This corresponds to a map

$$T_3 \rightarrow \bar{T}_3 = (1 - 2\Delta)T_3, \quad T_{1,2} \rightarrow \bar{T}_{1,2} = T_{1,2} \quad (20)$$

into the Cartesian basis of  $O(3)_\Delta$  and

$$[\bar{T}_i, \bar{T}_j] = \bar{\epsilon}_{ijk} \bar{T}_k, \quad (21)$$

$$\bar{\epsilon}_{ijk} = \begin{cases} \pm(1 - 2\Delta)^{-1} & \text{for } i(j) = 1(2), k = 3 \\ \pm(1 - 2\Delta) & \text{for } i \text{ or } j = 3, k = 1 \text{ or } 2 \\ 0 & \text{else} \end{cases} \quad (22)$$

and  $\bar{\epsilon}_{ijk} \rightarrow \epsilon_{ijk}$  with  $\Delta \rightarrow 0$ . Using  $\bar{\epsilon}_{ijk}$  from eq. (22) to write down  $\bar{T}$  in  $O(3)_\Delta$  is equivalent to rescale  $O(3)$  vectors

$$\mathbf{x} \rightarrow \mathbf{x}' = (x_1, x_2, x'_3 = \{1 - 2\Delta\}x_3) \quad (23)$$

with the result

$$\begin{aligned}
T'_1 &= \mathbf{e}'_1(x_2 p_3 - x_3 p_2) = T_1 \\
T'_2 &= \mathbf{e}'_2(x_3 p_1 - x_1 p_3) = T_2 \\
T'_3 &= \mathbf{e}'_3(x_1 p_2 - x_2 p_1) = (1 - 2\Delta) T_3.
\end{aligned} \tag{24}$$

For a classical free particle, eq. (23) produces a scalar potential  $U$ ,

$$L \rightarrow L' = \frac{1}{2m} p'^2 = L - U \tag{25}$$

$$U = -\frac{2\Delta}{m} p_3^2 + \dots, \tag{26}$$

which contains a vector potential  $\mathbf{A}$ . In the present case we have

$$U = -\mathbf{A} \cdot \mathbf{v} \tag{27}$$

and  $\mathbf{A} \equiv (0, 0, A_3)$  corresponds to a magnetic field

$$\mathbf{B} = \mathbf{e}_1 \partial_2 A_3 - \mathbf{e}_2 \partial_1 A_3. \tag{28}$$

Note that for a constant  $\mathbf{A}$  the deformation parameter becomes momentum dependent,

$$A_3 = 2\Delta_c = 2\Delta p_3, \tag{29}$$

where  $\Delta_c = \Delta p$  enters eq. (19) and subsequent definitions of  $O(3)_{\Delta_c}$  quantities. The classical free  $\Delta$ -particle is thus a particle in a magnetic field  $\mathbf{B}$  which is in the  $(x_1, x_2)$ -plane, while  $\mathbf{A}$  breaks  $O(3)$  explicitly and diverges with  $\Delta \rightarrow \frac{1}{2}$ . The limits are now:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \quad \text{at} \quad \Delta = 0, \tag{30}$$

$$SU(2)_L \times U(1)_Y \quad \text{plus potential } U \quad \text{at} \quad \Delta = \frac{1}{2}, \tag{31}$$

where  $U$  is the desired augmentation of  $L$  in eq. (25).



A general vector potential with components  $A_i = 2\Delta_i p_i = 2\Delta_i$  yields

$$L' = \frac{1}{2m}(p^2 + 4\mathbf{\Delta p}) \quad (32)$$

and corresponds to a rescaling

$$x_i \rightarrow x'_i = (1 - 2\Delta_i)x_i, \quad (33)$$

where we dropped the index  $c$ .

### 3. Connection with q-Deformation

The most studied generalization of  $SU(2)$  is the Drinfeld-Jimbo  $SU(2)_q$  algebra, given by

$$[\bar{T}_0, \bar{T}_\pm] = \pm \bar{T}_\pm, \quad [\bar{T}_+, \bar{T}_-] = [\bar{T}_0]_{q^2}, \quad (34)$$

where  $[x]_{q^2} \equiv (q^{2x} - q^{-2x})/(q^2 - q^{-2})$  and  $q \rightarrow 1$  yields  $SU(2)$ . The deformation (34) succesfully describes e.g. the anisotropic Heisenberg model. Comultiplication rules and many details of its representation theory are known [7].

Following Curtright and Zachos [8], we can find expressions  $\bar{T}_\pm$  as functions of  $SU(2)$  generators with the Ansatz

$$\begin{aligned} \bar{T}_+ \bar{T}_- &\equiv \frac{1}{q + 1/q} \cdot \left\{ [T]_q [T + 1]_q - [\bar{T}_0]_q [\bar{T}_0 - 1]_q \right\} \\ \bar{T}_- \bar{T}_+ &\equiv \frac{1}{q + 1/q} \cdot \left\{ [T]_q [T + 1]_q - [\bar{T}_0]_q [\bar{T}_0 + 1]_q \right\}. \end{aligned} \quad (35)$$

The Casimir is

$$C_q = 2T_+ T_- + [\bar{T}_0]_q [\bar{T}_0 - 1]_q = 2T_- T_+ + [\bar{T}_0]_q [\bar{T}_0 + 1]_q = [T]_q [T + 1]_q. \quad (36)$$

From eq. (18) we have  $2T_+ T_- = (T + T_0)(1 + T - T_0)$  and  $2T_- T_+ = (T - T_0)(1 + T + T_0)$  and with the  $q$ -analogues and eq. (35) we arrive at

$$\bar{T}_+ = \sqrt{\frac{2}{q + 1/q} \cdot \frac{[T + T_0]_q [1 + T - T_0]_q}{(T + T_0)(1 + T - T_0)}} T_+,$$

$$\bar{T}_- = \sqrt{\frac{2}{q+1/q} \cdot \frac{[T-T_0]_q [1+T+T_0]_q}{(T-T_0)(1+T+T_0)}} T_-, \quad (37)$$

and, from eq. (34),  $\bar{T}_0 = T_0$ . Inserting the  $T = \frac{1}{2}$  representation into eq. (37) gives

$$\bar{T}_\pm = \alpha T_\pm, \quad \alpha = \sqrt{2/(q+1/q)}. \quad (38)$$

In the cartesian basis we therefore have

$$\bar{T}_3 = T_3, \quad \bar{T}_{1,2} = \alpha T_{1,2} \quad (39)$$

and the commutators

$$[\bar{T}_1, \bar{T}_2] = i\alpha^2 T_3, \quad [T_3, \bar{T}_1] = i\bar{T}_2, \quad [\bar{T}_2, T_3] = i\bar{T}_1, \quad (40)$$

correspond to a rescaling

$$\mathbf{x} \rightarrow \mathbf{x}' = (\alpha x_1, \alpha x_2, x_3). \quad (41)$$

We identify

$$1 - 2\Delta = \sqrt{\frac{2}{q+1/q}} \quad (42)$$

and eqn. (32) and (33) yield a potential  $\mathbf{A} = (A_1, A_2, 0)$ , which corresponds to a field  $\mathbf{B}$  in  $x_3$ -direction. This agrees with the interpretation in [9]. Note that the classical limit  $q \rightarrow 1$  corresponds to  $\Delta \rightarrow 0$  in eq. (42). The second limit eq. (31)  $\Delta \rightarrow \frac{1}{2}$  corresponds to  $q \rightarrow \infty$ . Note the symmetry  $q \rightarrow q^{-1}$ .

We rotate vectors  $\mathbf{x}$  to  $\hat{\mathbf{x}} = R \mathbf{x}$ , where

$$R = \begin{pmatrix} r^2 & -r^2 & -r \\ -r^2 & r^2 & -r \\ r & r & 0 \end{pmatrix} \quad (43)$$

and  $r = \sin(\pi/4)$ . After applying the map eq. (41) they take the form

$$\begin{aligned} \hat{x}'_1 &= \alpha r^2 x_1 - \alpha r^2 x_2 - r x_3 \\ \hat{x}'_2 &= -\alpha r^2 x_1 + \alpha r^2 x_2 - r x_3 \\ \hat{x}'_3 &= \alpha r(x_1 + x_2) = \alpha(\hat{x}_1 + \hat{x}_2) \end{aligned} \quad (44)$$

and with eq. (42) the third component satisfies eq. (7) again.

We note that only  $\bar{T}_0$  is fixed in eq. (7) while  $\bar{T}_\pm$  are only required to close the algebra and yield the classical limit. There is also some freedom in normalizing the structure constants  $\bar{\epsilon}$  or commutators, namely  $[\bar{T}_+, \bar{T}_-] = \frac{1}{2}[2\bar{T}_0]_q$  does not exhibit the deformation in the generators of the  $T = \frac{1}{2}$  representation [8].

## 4. Change of Topology

We want to remark that there is actually more freedom in the dynamics of the classical particle than considered in sections 1 and 2. Eq. (7) fixes the generators of the Cartan subalgebra while ‘charged operators’  $\bar{T}_\pm$  are only required to close  $SU(2)_\Delta$  and yield ordinary  $SU(2)$  when  $\Delta \rightarrow 0$ . The effect of the symmetry breaking parameter being nonzero can in principle be more drastic than  $GL(2)$  transformations on the ordinary representations, namely the topology might be altered in the following sense.

Eq. (38) directly shows that  $\bar{T}_\pm$  create the vacuum out of lowest and highest weight states: the ordinary normalization of states  $\bar{T}_\pm|t, t_0\rangle = \sqrt{(t \mp t_0)(t \pm t_0 + 1)}|t, t_0 \pm 1\rangle$  has become

$$\bar{T}_\pm|t, t_0\rangle = \alpha\sqrt{[t \mp t_0]_q[t \pm t_0 + 1]_q}|t, t_0 \pm 1\rangle, \quad (45)$$

[10], and  $\bar{T}_\pm|t, t_0 \pm 1\rangle = 0$ .

The Ansatz eq. (37) can however be relaxed by adding to  $\bar{T}_+\bar{T}_-$  and  $\bar{T}_-\bar{T}_+$  a term  $f(q) = a(q)[\bar{T}_+ + \bar{T}_- + a(q)]$ , i.e. adding  $a(q)$  to  $\bar{T}_+$  and  $\bar{T}_-$ . In the difference eq. (34),  $f(q)$  vanishes. For  $a(q)$  a number, the new representation would for example read

$$\tilde{T}_+ = \bar{T}_+ + a(q) = \begin{pmatrix} a & \alpha \\ 0 & a \end{pmatrix}$$

$$\tilde{T}_- = \bar{T}_- + a(q) = \begin{pmatrix} a & 0 \\ \alpha & a \end{pmatrix}. \quad (46)$$

If  $\bar{T}_\pm + a = \tilde{T}_1 \pm i\tilde{T}_2$  one sees that this would correspond to

$$\mathbf{x} \rightarrow \mathbf{x}' = \mathbf{x} + a \quad (47)$$

and  $a$  is not a parameter of  $O(3)$ -geometry, but gives a coordinate independent term (constraint) in the equations of motion.

In a gauge theory, one would expect anomalies to appear in this case and if we naively enter the sum of triangle diagrams with the ‘anomalous couplings’ eq. (46), we get a contribution proportional to  $\text{Tr}\tilde{T}_i = 2a$ .

## 5. Summary

On the basis of chiral  $U(2)$ , we defined a continuous transition from the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representation of the Left-Right model to the  $SU(2)_L \times U(1)_Y$  representation of the minimal Standard Model. The transition parameter  $\Delta$  is the splitting of hypercharges of right-handed up- and down-type fermions in the  $T = \frac{1}{2}$  representation. Classically,  $\Delta$  is proportional to a (iso)magnetic field which vanishes in the LR and diverges in the limit of the SM.  $\Delta$  is therefore a measure of explicit breaking of chiral  $U(2)$ . The same happens in  $q$ -deformation such that we get  $\Delta = \frac{1}{2} \left[ 1 - \sqrt{2/(q + q^{-1})} \right]$ .

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